This question paper contains 4+2 printed pages]

HPAS (Main)—2017

STATISTICS

Paper II

Time : 3 *Hours* *Maximum Marks* : 100

Note :— Attempt Question No. **1** which is compulsory and other *four* questions from the rest. *All* questions carry equal marks. Symbols used in questions have their usual meanings.

1. (*a*) State and prove Nayman-Pearson's lemma for testing a simple hypothesis against a simple alternative hypothesis. 6

(*b*) Let p be the probability that a coin will fall head in a single toss in order to test $H_0 : p = \frac{1}{2}$ against $H_1 : p = \frac{3}{4}$. The coin is tossed 5 times and H_0 is rejected if more than 3 heads are obtained. Find the probability of type Ist error and power of the test. 6

P.T.O.

(*c*) Let $x_1, x_2, \ldots\ldots, x_n$ be a random sample of n observations from $N(\mu, \sigma^2)$. Find BCR and power of the test in testing the significance of $H_0 : \mu = \mu_0$ against $H_1 : \mu = \mu_1$. Assume that σ^2 is known. 8

2. (*a*) Define likelihood ratio test. Let $x_1, x_2, \ldots\ldots, x_n$ be random sample of n observations from a population having p.d.f. :

$$f(x) = \begin{cases} e^{-(x-\theta)}, & \theta \le x \le \infty \\ 0 \quad , & \text{otherwise} \end{cases}.$$

Find UMP critical region to test $H_0 : \theta = \theta_0$ against $H_1 : \theta \neq \theta_0$. 10

(*b*) Explain the terms—multiple and partial correlation. Using Yule's notations R for multiple and r for partial correlation, in a trivariate distribution given :

$\sigma_1 = 2, \sigma_2 = \sigma_3 = 3, r_{12} = 0.7, r_{23} = r_{31} = 0.5$.

Find :

(*i*) $r_{23.1}$

(*ii*) $R_{1.23}$

(*iii*) $\sigma_{1.23}$. 10

3. (*a*) Explain the term 'distribution free methods'. Derive the run test, stating clearly the assumptions made. In a win-lose record of a certain basketball team for their last 50 consecutive games was as follows :

W W W W W W L W W W W W W L W L W W W L L W W W W L W W W L L W W W W W W L L W W L L L W W L W W W.

Apply run test to test the randomness of wins and loses. 10

(*b*) Let $x_1, x_2, \ldots\ldots, x_m$ be a random sample of m observations from $N(\mu, \sigma^2)$. Deduce SPRT statistic to test the significance of $H_0 : \mu = \mu_0$ against $H_1 : \mu = \mu_1$. Also find its OC and ASN functions. 10

P.T.O.

4. (*a*) In a simple random sampling without replacement prove that sample mean is unbiased estimate of its population mean. Also derive the expression for its variance. 10

(*b*) Explain the ratio method of estimation. If y_i, x_i are measured on each unit of a simple random sample of size n, assumed large. Prove that the mean square error (MSE) and variane of $\hat{R} = \bar{y}/\bar{x}$ are each approximately :

$$\text{MSE}(\hat{R}) \doteq \text{V}(\hat{R}) \doteq \frac{1-f}{n\bar{X}^2} \frac{\sum_{i=1}^{N}(y_i - Rx_i)^2}{N-1}$$

where $R = \bar{Y}/\bar{X}$ is the ratio of population means and $f = \frac{n}{N}$ is sampling fraction. 10

5. (*a*) What assumptions need to be made while using the technique of analysis of variance. Obtain ANOVA table for one way classified data. Derive the expression for critical difference if the treatments show significance effect. 10

(*b*) Derive the expressions for expectations of treatment sum of square (SST), variety sum of square (SSV) and error sum of squares (SSE) in case of analysis of variance for two way classified data. 10

6. (*a*) Give the layout and analysis of completely randomised design (CRD). In what situations it is most preferably used ? 10

(*b*) Explain various transformations of variate generally used to stabilize variance in case of violation of assumptions considered in ANOVA technique. 10

7. (*a*) What do you mean by resolvable and affine resolvable BIBD ? Prove that for a resolvable BIBD with parameters v, b, r, k and λ :

(*i*) $v_r = bk$

(*ii*) $b \geq v + r - 1$. 10

(*b*) Explain complete and partial confounding in factorial experiments. How will you confound interaction ABCD in 2^3-factorial experiment ? 10

8. (*a*) Differentiate between multistage and multiphase sampling methods of estimation. Establish a relationship between mean square error, variance and bias of the estimate of population mean. 10

(*b*) What are the steps of large scale sample surveys ? Obtain the expression for size of sample for specified precision in a large survey. 10